

# Distortions of the Harrison-Zel'dovich spectrum from the QCD transition

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## Abstract

We investigate the effect of the cosmological QCD transition on the evolution of primordial density perturbations. If the phase transition is first order, the sound speed vanishes during the transition and density perturbations fall freely. The Harrison-Zel'dovich spectrum of density fluctuations develops large peaks on scales below the Hubble radius at the transition. These peaks above the primordial spectrum grow with wavenumber and produce cold dark matter clumps of masses less than  $10^{-10} M_\odot$ . At the horizon scale the amplification of overdensities is of order unity, and thus no  $1 M_\odot$  black hole production is possible for a COBE normalized scale-invariant spectrum.

**Introduction.** The QCD transition from a hot plasma of deconfined quarks and gluons to a hot gas of hadrons happens in the early Universe at a temperature  $T_\star$ . Lattice QCD results for the physical values of the quark masses indicate that the QCD phase transition is of first order [1] and takes place at  $T_\star \sim 150$  MeV [2]. The QCD transition may have important cosmological consequences: it could lead to inhomogeneous nucleosynthesis [3], produce dark matter in the form of strangelets [4], or give rise to gravitational waves. These effects have in common that their scale is set by the mean bubble nucleation distance  $R_{\text{nucl}}$  and not by the Hubble scale  $R_H$ . It has turned out that  $R_{\text{nucl}} \ll R_H$ , mainly because the surface tension is very small [5]. In contrast, we study the scales  $\lambda \gg R_{\text{nucl}}$ .

Cosmological density perturbations are affected by the QCD transition at scales  $\lambda \lesssim R_H$  [6, 7], where  $R_H \sim m_P/T_\star^2 \sim 10^4$  m. For a first order

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QCD phase transition the deconfined and confined phases can coexist at the coexistence temperature  $T_\star$  at fixed pressure  $p_\star = p(T_\star)$ . During this coexistence period pressure gradients, and thus the sound speed, vanish for wavelengths  $\lambda \gg R_{\text{nuc}}$ . The vanishing sound speed gives rise to large peaks and dips above the primordial spectrum of density perturbations. These peaks grow at most linearly with wavenumber. We show that the formation of black holes at the horizon scale is impossible for standard inflationary scenarios, in contrast to recent claims [3, 8]. We predict clumps with  $M < 10^{-10} M_\odot$  in cold dark matter (CDM), if the CDM is kinetically decoupled at the QCD transition.

**The cosmological QCD transition.** The QCD phase transition starts with a short period ( $10^{-4} t_H$ ) of tiny supercooling,  $1 - T_{\text{sc}}/T_\star \sim 10^{-3}$ . When  $T$  reaches  $T_{\text{sc}}$ , bubbles nucleate at mean distances  $R_{\text{nuc}} \lesssim 2 \text{ cm}$  [11]. The bubbles grow most probably by weak deflagration [10]. The released energy is transported into the deconfined phase by shock waves, which reheat the deconfined phase to  $T_\star$  within  $10^{-6} t_H$ . The entropy production during this very short period is  $dS/S \sim 10^{-3}$ . Further bubble nucleation in the reheated quark-gluon phase is prohibited after this first  $10^{-4} t_H$ . Thereafter, the bubbles grow adiabatically due to the expansion of the Universe. The transition completes after  $10^{-1} t_H$ .

During this equilibrium evolution the rate at which the quark-gluon phase can be converted into the hadron phase and vice versa,  $\Gamma_{\text{conv}}$ , must be compared to the Hubble rate  $H$  and to the wave number  $k^{\text{phys}}$  of the considered density perturbation.  $\Gamma_{\text{conv}}$  cannot enormously differ from the typical QCD scale  $\text{fm}^{-1}$ , while  $H \approx (10^4 \text{ m})^{-1}$ , therefore  $H/\Gamma_{\text{conv}}$  and  $k^{\text{phys}}/\Gamma_{\text{conv}}$  are of order  $10^{-19}$ . This means that the QCD relaxation times are incredibly short and that the 2-phase system is very close to equilibrium. Since all interaction rates  $\Gamma_{\text{strong,elweak}} \gg H$ , all particles are in chemical and thermal equilibrium. At scales  $\lambda > R_{\text{nuc}}$  we may describe photons, leptons, and the QCD matter by a single radiation fluid with equation of state  $p = p(\rho)$ .

During this reversible coexistence period (isentropic)  $T$  and  $p$  are fixed, while  $\rho$  can vary, therefore

$$c_s^2 \equiv \left( \frac{\partial p}{\partial \rho} \right)_{\text{isentropic}} = 0 . \quad (1)$$

In [12] it was claimed that the isentropic condition is not consistent. This is wrong, because they apply non-relativistic ( $v \ll 1, p \ll \rho$ ) hydrodynamical equations to relativistic wave fronts in the radiation fluid. Moreover, they claim that sound propagation keeps the energy fraction in each phase fixed. This is incompatible with the equilibrium situation described above.

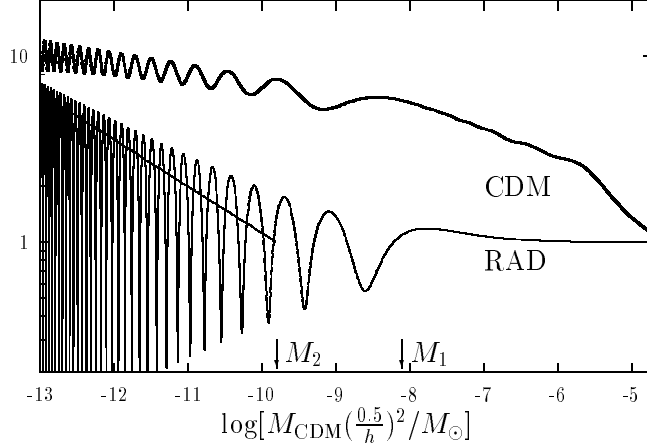


Figure 1: The modifications of the CDM density contrast  $|\delta_{\text{CDM}}|(T_\star/10)$  and of the radiation fluid amplitude  $A_{\text{RAD}}$  due to the QCD transition (lattice QCD fit). Both quantities are normalized to the pure Harrison-Zel'dovich radiation amplitude. On the horizontal axis the wavenumber  $k$  is represented by the CDM mass contained in a sphere of radius  $\pi/k$ .

The thermodynamics of the cosmological QCD transition can be studied in lattice QCD, because the baryochemical potential is negligible in the early Universe, i.e.  $\mu_b/T_\star \sim 10^{-8}$ . We fit the equation of state from lattice QCD [9]. In addition we use the bag model to illustrate the origin of the large peaks in the density spectrum.

**The evolution of density perturbations.** During an inflationary epoch in the early Universe density perturbations  $\delta\rho$  have been generated with a scale-invariant Harrison-Zel'dovich (HZ) spectrum. During the radiation dominated era subhorizon ( $k^{\text{phys}} \gg H$ ) density perturbations oscillate as acoustic waves, supported by the pressure  $\delta p = c_s^2 \delta\rho$ . It is useful to study the dimensionless density contrast  $\delta \equiv \delta\rho/\rho$  as a function of conformal time and comoving wave number. The subhorizon equation of motion reads

$$\delta'' + c_s^2 k^2 \delta = 0, \quad (2)$$

if we assume that the phase transition is short compared to the Hubble time. From the lattice QCD equation of state the period of vanishing  $c_s^2$  lasts for  $0.1t_H$ , in the bag model it lasts for  $0.3t_H$ .

To illustrate the mechanism of generating large peaks above the HZ spectrum we discuss the subhorizon evolution of the density contrast  $\delta$  in the bag model [7]. Before and after the QCD transition  $c_s^2 = 1/3$  and the density contrast oscillates with constant amplitudes  $A^{\text{in}}$  resp.  $A^{\text{out}}$ . At the coexistence temperature  $T_\star$  the pressure gradient, i.e. the restoring force in in Eq. (2),

vanishes and the sound speed (1) drops to zero. Thus  $\delta$  will grow or decrease linearly, depending on the fluid velocity at the moment when the phase transition starts. This produces large peaks in the spectrum which grow linearly with the wave number, i.e.  $A^{\text{out}}/A^{\text{in}}|_{\text{peaks}} \approx k/k_1$ . The radiation in a Hubble volume at  $T_*$  has the mass  $M_{\text{RAD}}(R_H) \sim 1M_\odot$ . The CDM mass is related to the radiation mass by  $M_{\text{CDM}} = a_*/a_{\text{eq}}M_{\text{RAD}} \sim 10^{-8}M_{\text{RAD}}$ . CDM falls into the gravitational potential wells of the radiation fluid during the coexistence regime.

Fig. 1 shows the final spectrum for the density contrast of the radiation fluid and of CDM. It has been obtained from numerical integration of the fully general relativistic equations of motion and the lattice QCD equation of state [6]. The mass  $M_1$  corresponds to the wave number  $k_1$  of the bag model analysis.  $M_1$  coincides with the mass inside the Hubble horizon. The mass  $M_2$  stems from a WKB analysis with the lattice QCD equation of state, which shows that  $A^{\text{out}}/A^{\text{in}}|_{\text{peaks}} \approx (k/k_2)^{3/4}$ . We conclude that at the horizon scale the modification of the HZ spectrum is mild, whereas for scales much smaller than the horizon big amplifications are predicted. Without tilt in the COBE normalized spectrum the density contrast grows nonlinear for  $k^{\text{phys}}/H \gtrsim 10^4$  resp.  $10^6$  for the bag model resp. lattice QCD equation of state.

**Implications of the large peaks.** It was suggested that the QCD transition could lead to the formation of  $1M_\odot$  black holes that could account for all missing (dark) matter [3, 8], i.e. all the radiation mass within one Hubble horizon should collapse to a single black hole. In common inflationary scenarios the density contrast is of order  $10^{-4}$  on subhorizon scales before the QCD transition. Thus, our linear analysis applies. From Fig. 1 it is clear that there is only a small amplification of order unity at the horizon scale (around  $M_1$ ). Therefore, with COBE normalized scale-invariant spectrum (allowing tilts  $|n - 1| < 0.3$ ) it is impossible to form black holes at the QCD transition that are abundant enough to close the Universe.

At  $T \sim 1$  MeV the neutrinos decouple from the Hubble scale, from smaller scales they decouple at higher temperatures. During their decoupling they damp all fluctuations in the density contrast due to collisional damping. Thus, at the time of big bang nucleosynthesis all peaks of Fig. 1 are erased and the energy density on scales  $\lambda \lesssim R_H(1 \text{ MeV})$  is homogeneous.

Weakly interacting dark matter, like the neutralino, does not belong to our CDM, because it is not kinetically decoupled at the QCD transition. For our purpose the neutralino would belong to the radiation fluid and subhorizon perturbations in its density are damped during its kinetic decoupling.

However, collisional damping is irrelevant for other CDM like primordial black holes. Peaks in this CDM survive and grow logarithmically during the

radiation era. Shortly after equality they grow nonlinear and collapse by gravitational virialization to clumps of  $M < 10^{-10} M_{\odot}$ . Their size can be estimated to be a few AU. Whether such CDM clumps may be observable remains to be investigated.

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